PowerPoint to accompany

Introduction to MATLAB for Engineers, Third Edition

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Chapter 2
Numeric, Cell, and Structure Arrays

Vectors: To create a *row* vector, separate the elements by semicolons. Use square brackets. For example,

```
>>p = [3,7,9]
p =
    3    7    9
```

You can create a *column* vector by using the *transpose* notation (').

```
>>p = [3,7,9]'
p =
    3
    7
    9
```

You can also create a column vector by separating the elements by semicolons. For example,

```
>>g = [3;7;9]
g =
   3
   7
   9
```

You can create vectors by "appending" one vector to another.

For example, to create the row vector `u` whose first three columns contain the values of `r = [2,4,20]` and whose fourth, fifth, and sixth columns contain the values of `w = [9,-6,3]`, you type `u = [r,w]`. The result is the vector `u = [2,4,20,9,-6,3]`.

The colon operator (:) easily generates a large vector of regularly spaced elements. Parentheses are not needed but can be used for clarity. Do not use square brackets.

Typing

```
>>x = m:q:n
```

or

```
>>x = (m:q:n)
```

creates a vector `x` of values with a spacing `q`. The first value is `m`. The last value is `n` if `m - n` is an integer multiple of `q`. If not, the last value is less than `n`.

For example, typing **`x = 0:2:8`** creates the vector `x = [0,2,4,6,8]`, whereas typing `x = 0:2:7` creates the vector `x = [0,2,4,6]`.

To create a row vector z consisting of the values from 5 to 8 in steps of 0.1, type `z = 5:0.1:8`.

If the increment q is omitted, it is presumed to be 1. Thus typing **`y = -3:2`** produces the vector `y = [-3,-2,-1,0,1,2]`.

The `linspace` command also creates a linearly spaced row vector, but instead you specify the number of values rather than the increment.

The syntax is **`linspace(x1,x2,n)`**, where **`x1` and `x2` are the lower and upper limits and `n` is the number of points.**

For example, **`linspace(5,8,31)`** is equivalent to `5:0.1:8`.

If `n` is omitted, the spacing is 1.

The `logspace` command creates an array of *logarithmically* spaced elements.

Its syntax is `logspace(a,b,n)`, where `n` is the number of points between 10^a and 10^b.

For example, `x = logspace(-1,1,4)` produces the vector `x = [0.1000, 0.4642, 2.1544, 10.000]`.

If `n` is omitted, the number of points defaults to 50.

Magnitude, Length, and Absolute Value of a Vector

Keep in mind the precise meaning of these terms when using MATLAB.

The `length` command gives the *number of elements* in the vector.

The *magnitude* of a vector **x** having elements x_1, x_2, …, x_n is a scalar, given by $\sqrt{}(x_1^2 + x_2^2 + \ldots + x_n^2)$, and is the same as the vector's geometric length.

The *absolute value* of a vector **x** is a vector whose elements are the absolute values of the elements of **x**.

For example, if `x = [2,-4,5]`,

- its length is 3; (computed from `length(x)`)
- its magnitude is $\sqrt{[2^2 + (-4)^2 + 5^2]} = 6.7082$; (computed from `sqrt(x'*x)`)
- its absolute value is `[2,4,5]` (computed from `abs(x)`).

Matrices

A matrix has multiple rows and columns. For example, the matrix

$$\mathbf{M} = \begin{bmatrix} 2 & 4 & 10 \\ 16 & 3 & 7 \\ 8 & 4 & 9 \\ 3 & 12 & 15 \end{bmatrix}$$

has four rows and three columns.

Vectors are special cases of matrices having one row or one column.

Creating Matrices

If the matrix is small you can type it row by row, separating the *elements* in a given row with *spaces* or *commas* and separating the *rows* with semicolons. For example, typing

```
>>A = [2,4,10;16,3,7];
```

creates the following matrix:

$$\mathbf{A} = \begin{bmatrix} 2 & 4 & 10 \\ 16 & 3 & 7 \end{bmatrix}$$

Remember, spaces or commas separate elements in different *columns*, whereas semicolons separate elements in different *rows*.

Creating Matrices from Vectors

Suppose `a = [1,3,5]` and `b = [7,9,11]` (row vectors). Note the difference between the results given by `[a b]` and `[a;b]` in the following session:

```
>>c = [a b];
c =
   1   3   5   7   9   11
>>D = [a;b]
D =
   1   3   5
   7   9   11
```

You need not use symbols to create a new array. For example, you can type

```
>> D = [[1,3,5];[7,9,11]];
```

Array Addressing

The colon operator selects individual elements, rows, columns, or "subarrays" of arrays. Here are some examples:

- v(:) represents all the row or column elements of the vector v.

- v(2:5) represents the second through fifth elements; that is v(2), v(3), v(4), v(5).

Array Addressing, continued

- A(:,3) denotes all the elements in the third column of the matrix A.
- A(:,2:5) denotes all the elements in the second through fifth columns of A.
- A(2:3,1:3) denotes all the elements in the second and third rows that are also in the first through third columns.
- v = A(:) creates a vector v consisting of all the columns of A stacked from first to last.
- A(end,:) denotes the last row in A, and A(:,end) denotes the last column.

You can use array indices to extract a smaller array from another array. For example, if you first create the array **B**

$$\mathbf{B} = \begin{bmatrix} 2 & 4 & 10 & 13 \\ 16 & 3 & 7 & 18 \\ 8 & 4 & 9 & 25 \\ 3 & 12 & 15 & 17 \end{bmatrix}$$

then type `C = B(2:3,1:3),` you can produce the following array:

$$\mathbf{C} = \begin{bmatrix} 16 & 3 & 7 \\ 8 & 4 & 9 \end{bmatrix}$$

Additional Array Functions

`[u,v,w] = find(A)`	Computes the arrays `u` and `v`, containing the row and column indices of the nonzero elements of the matrix `A`, and the array `w`, containing the values of the nonzero elements. The array `w` may be omitted.
`length(A)`	Computes either the number of elements of `A` if `A` is a vector or the largest value of `m` or `n` if `A` is an $m \times n$ matrix.

Additional Array Functions

`max(A)`	Returns the algebraically largest element in `A` if **`A`** is a vector. Returns a row vector containing the largest elements in each column if **`A`** is a matrix. If any of the elements are complex, `max(A)` returns the elements that have the largest magnitudes.

Additional Array Functions

`[x,k] = max(A)`	Similar to `max(A)` but stores the maximum values in the row vector `x` and their indices in the row vector `k`.
`min(A)` and `[x,k] = min(A)`	Like `max` but returns minimum values.

Additional Array Functions

`size(A)`	Returns a row vector `[m n]` containing the sizes of the *m* x *n* array `A`.
`sort(A)`	Sorts each column of the array `A` in ascending order and returns an array the same size as `A`.
`sum(A)`	Sums the elements in each column of the array `A` and returns a row vector containing the sums.

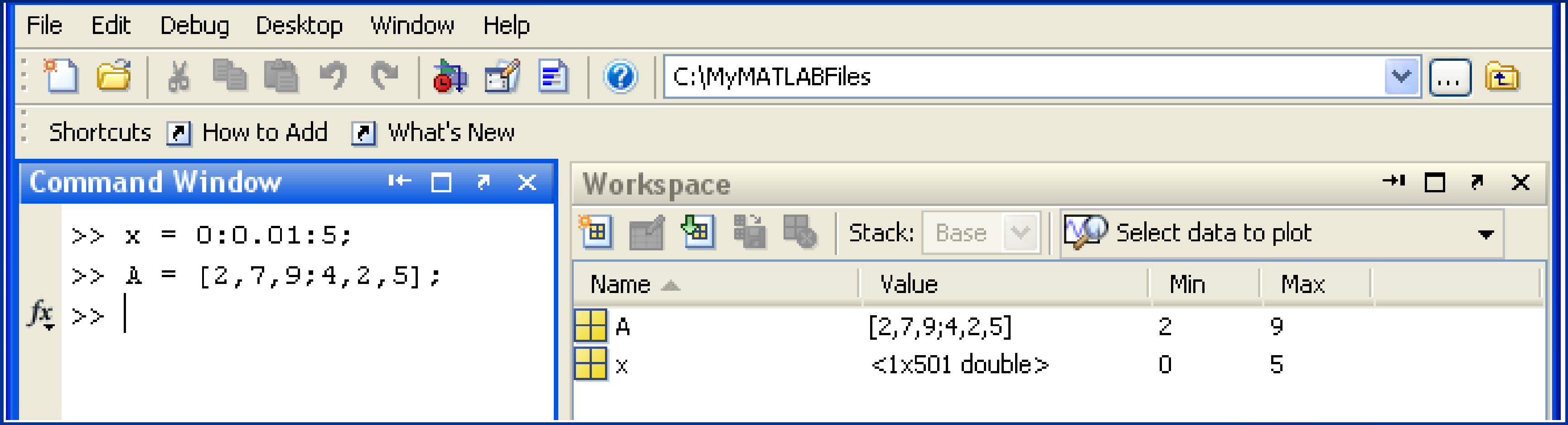
The function `size(A)` returns a row vector `[m n]` containing the sizes of the $m \times n$ array **A**. The `length(A)` function computes either the number of elements of **A** if `A` is a vector or the largest value of m or n if **A** is an $m \times n$ matrix.

For example, if

$$\mathbf{A} = \begin{bmatrix} 6 & 2 \\ -10 & -5 \\ 3 & 0 \end{bmatrix}$$

then `max(A)` returns the vector `[6,2]`; `min(A)` returns the vector `[-10, -5]`; `size(A)` returns `[3, 2]`; and `length(A)` returns `3`.

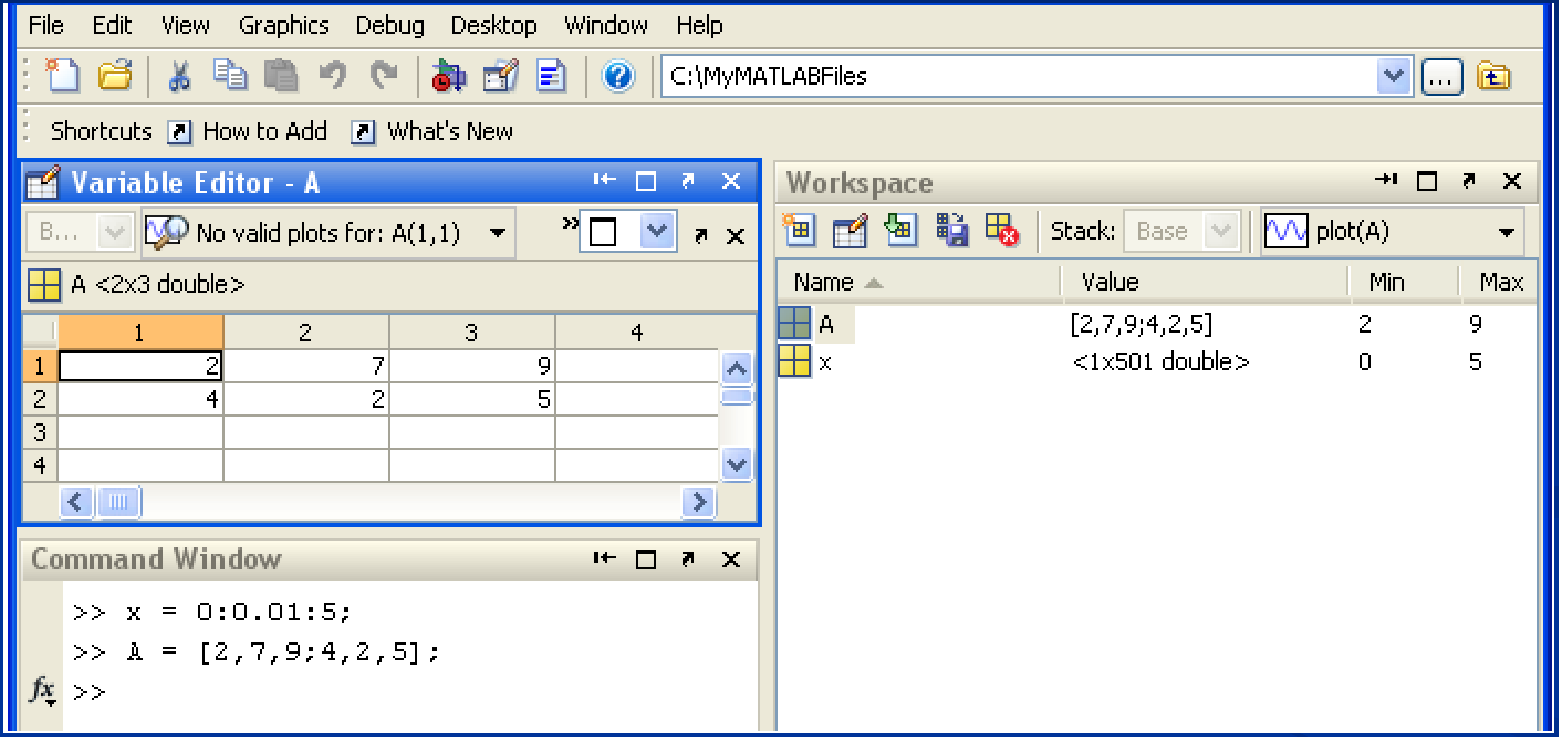
The Workspace Browser.



```
>> x = 0:0.01:5;
>> A = [2,7,9;4,2,5];
>> 
```

Name	Value	Min	Max
A	[2,7,9;4,2,5]	2	9
x	<1x501 double>	0	5

The Variable Editor.


File Edit View Graphics Debug Desktop Window Help
C:\MyMATLABFiles
Shortcuts How to Add What's New
Variable Editor - A
No valid plots for: A(1,1)
A <2x3 double>
Workspace
Stack: Base
plot(A)
Command Window

	1	2	3	4
1	2	7	9	
2	4	2	5	
3				
4				

Name	Value	Min	Max
A	[2,7,9;4,2,5]	2	9
x	<1x501 double>	0	5

```
>> x = 0:0.01:5;
>> A = [2,7,9;4,2,5];
>>
```

Multidimensional Arrays

Consist of two-dimensional matrices “layered” to produce a third dimension. Each “layer” is called a *page*.

`cat(n,A,B,C, ...)` Creates a new array by concatenating the arrays `A`,`B`,`C`, and so on along the dimension `n`.

Array Addition and Subtraction

For example:

$$\begin{bmatrix} 6 & -2 \\ 10 & 3 \end{bmatrix} + \begin{bmatrix} 9 & 8 \\ -12 & 14 \end{bmatrix} = \begin{bmatrix} 15 & 6 \\ -2 & 17 \end{bmatrix}$$

Array subtraction is performed in a similar way.
The addition shown in equation 2.3–1 is performed in MATLAB as follows:

```
>>A = [6,-2;10,3];
>>B = [9,8;-12,14]
>>A+B
ans =
    15      6
    -2      17
```

Multiplication: Multiplying a matrix **A** by a scalar *w* produces a matrix whose elements are the elements of **A** multiplied by *w*. For example:

$$3\begin{bmatrix} 2 & 9 \\ 5 & -7 \end{bmatrix} = \begin{bmatrix} 6 & 27 \\ 15 & -21 \end{bmatrix}$$

This multiplication is performed in MATLAB as follows:

```
>>A = [2, 9; 5,-7];
>>3*A
ans =
     6   27
    15  -21
```

Multiplication of an array by a scalar is easily defined and easily carried out.

However, multiplication of two *arrays* is not so straightforward.

MATLAB uses two definitions of multiplication:

(1) array multiplication (also called *element-by-element* multiplication), and

(2) matrix multiplication.

Division and exponentiation must also be carefully defined when you are dealing with operations between two arrays.

MATLAB has two forms of arithmetic operations on arrays. Next we introduce one form, called *array* operations, which are also called *element-by-element* operations. Then we will introduce *matrix* operations. Each form has its own applications.

Division and exponentiation must also be carefully defined when you are dealing with operations between two arrays.

Element-by-element operations: Important

Symbol	Operation	Form	Examples
`+`	Scalar-array addition	`A + b`	`[6,3]+2=[8,5]`
`-`	Scalar-array subtraction	`A - b`	`[8,3]-5=[3,-2]`
`+`	Array addition	`A + B`	`[6,5]+[4,8]=[10,13]`
`-`	Array subtraction	`A - B`	`[6,5]-[4,8]=[2,-3]`
`.*`	Array multiplication	`A.*B`	`[3,5].*[4,8]=[12,40]`
`./`	Array right division	`A./B`	`[2,5]./[4,8]=[2/4,5/8]`
`.\`	Array left division	`A.\B`	`[2,5].\[4,8]=[2\4,5\8]`
`.^`	Array exponentiation	`A.^B`	`[3,5].^2=[3^2,5^2]`
			`2.^[3,5]=[2^3,2^5]`
			`[3,5].^[2,4]=[3^2,5^4]`

Array or *Element-by-element* multiplication is defined only for arrays having the same size. The definition of the product `x.*y`, where `x` and `y` each have *n* elements, is

```
x.*y = [x(1)y(1), x(2)y(2), ... , x(n)y(n)]
```

if `x` and `y` are row vectors. For example, if

```
x = [2, 4, – 5], y = [– 7, 3, – 8]
```

then `z = x.*y` gives

z = [2(– 7), 4 (3), –5(–8)] = [–14, 12, 40]

If `x` and `y` are column vectors, the result of `x.*y` is a column vector. For example `z = (x').*(y')` gives

$$\mathbf{z} = \begin{bmatrix} 2(-7) \\ 4(3) \\ -5(-8) \end{bmatrix} = \begin{bmatrix} -14 \\ 12 \\ 40 \end{bmatrix}$$

Note that `x'` is a column vector with size 3 × 1 and thus does not have the same size as `y`, whose size is 1 × 3.

Thus for the vectors `x` and `y` the operations `x'.*y` and `y.*x'` are not defined in MATLAB and will generate an error message.

The array operations are performed between the elements in corresponding locations in the arrays. For example, the array multiplication operation `A.*B` results in a matrix `C` that has the same size as `A` and `B` and has the elements $c_{ij} = a_{ij} b_{ij}$. For example, if

$$\mathbf{A} = \begin{bmatrix} 11 & 5 \\ -9 & 4 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} -7 & 8 \\ 6 & 2 \end{bmatrix}$$

then `C = A.*B` gives this result:

$$\mathbf{C} = \begin{bmatrix} 11(-7) & 5(8) \\ -9(6) & 4(2) \end{bmatrix} = \begin{bmatrix} -77 & 40 \\ -54 & 8 \end{bmatrix}$$

The built-in MATLAB functions such as `sqrt(x)` and `exp(x)` automatically operate on array arguments to produce an array result the same size as the array argument `x`.

Thus these functions are said to be *vectorized* functions.

For example, in the following session the result `y` has the same size as the argument `x`.

```
>>x = [4, 16, 25];
>>y = sqrt(x)
y =
   2   4   5
```

However, when multiplying or dividing these functions, or when raising them to a power, you must use element-by-element operations if the arguments are arrays.

For example, to compute $z = (e^y \sin x) \cos^2 x$, you must type

```
z = exp(y).*sin(x).*(cos(x)).^2.
```

You will get an error message if the size of `x` is not the same as the size of `y`. The result `z` will have the same size as `x` and `y`.

Array Division

The definition of array division is similar to the definition of array multiplication except that the elements of one array are divided by the elements of the other array. Both arrays must have the same size. The symbol for array right division is ./. For example, if

```
x = [8, 12, 15]     y = [−2, 6, 5]
```

then `z = x./y` gives

z = [8/(–2), 12/6, 15/5] = [–4, 2, 3]

Also, if

$$\mathbf{A} = \begin{bmatrix} 24 & 20 \\ -9 & 4 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} -4 & 5 \\ 3 & 2 \end{bmatrix}$$

then `C = A./B` gives

$$\mathbf{C} = \begin{bmatrix} 24/(-4) & 20/5 \\ -9/3 & 4/2 \end{bmatrix} = \begin{bmatrix} -6 & 4 \\ -3 & 2 \end{bmatrix}$$

Array Exponentiation

MATLAB enables us not only to raise arrays to powers but also to raise scalars and arrays to *array* powers.

To perform exponentiation on an element-by-element basis, we must use the `.^` symbol.

For example, if `x = [3, 5, 8]`, then typing `x.^3` produces the array $[3^3, 5^3, 8^3] = [27, 125, 512]$.

We can raise a scalar to an array power. For example, if `p = [2, 4, 5]`, then typing `3.^p` produces the array $[3^2, 3^4, 3^5] = [9, 81, 243]$.

Remember that `.^` is a *single* symbol. The dot in `3.^p` is *not* a *decimal point* associated with the number 3. The following operations, with the value of `p` given here, are equivalent and give the correct answer:

```
3.^p
3.0.^p
3..^p
(3).^p
3.^[2,4,5]
```

Matrix-Matrix Multiplication

In the product of two matrices **AB**, the number of *columns* in **A** must equal the number of *rows* in **B**. The row-column multiplications form column vectors, and these column vectors form the matrix result. The product **AB** has the same number of *rows* as **A** and the same number of *columns* as **B**. For example,

$$\begin{bmatrix} 6 & -2 \\ 10 & 3 \\ 4 & 7 \end{bmatrix} \begin{bmatrix} 9 & 8 \\ -5 & 12 \end{bmatrix} = \begin{bmatrix} (6)(9) + (-2)(-5) & (6)(8) + (-2)(12) \\ (10)(9) + (3)(-5) & (10)(8) + (3)(12) \\ (4)(9) + (7)(-5) & (4)(8) + (7)(12) \end{bmatrix}$$

$$= \begin{bmatrix} 64 & 24 \\ 75 & 116 \\ 1 & 116 \end{bmatrix}$$

Matrix Left Division and Linear Algebraic Equations

$$6x + 12y + 4z = 70$$
$$7x - 2y + 3z = 5$$
$$2x + 8y - 9z = 64$$

```
>>A = [6,12,4;7,-2,3;2,8,-9];
>>B = [70;5;64];
>>Solution = A\B
Solution =
     3
     5
     -2
```

The solution is $x = 3$, $y = 5$, and $z = -2$.